length. Taking into account the additional internal energy relaxation mechanism leads to a decrease in the binding coefficient and an increase in the shear angle in comparison with the model of generalized thermomechanics. With a decrease in the internal energy relaxation time, the binding coefficient decreases and the shear angle increases. The difference in the binding coefficient may be observed for $\xi > 1.0$ while the difference in the shear angle may be observed for $\xi > 0.1$.

Thus, taking thermal memory into account is similar to taking relaxation of thermal flow into account [5] and taking rate of change of temperature into account [6], necessary for large frequencies or for small wavelengths. The relations we have obtained for velocities and damping of thermoelastic waves may find application in the experimental verification of thermoelastic models for the establishment of explicit expressions for heat flow and internal energy relaxation functions.

NOTATION

z, coordinate; t, time; ϑ, temperature; $\tilde{\alpha}(t)$, thermal flow relaxation function; $\tilde{\beta}(t)$, internal energy relaxation function; $\tilde{\gamma}(t)$, function of temperature relaxation of stresses; c_V, volumetric heat capacity; κ_i, linearization coefficients; ρ, density; u, displacement; ω, wave frequency; e, binding (compendency) coefficient; τ_q, thermal flow relaxation time; τ_e, internal energy relaxation time; τ_σ, time of temperature relaxation of stresses; λ, thermal conductivity; η, wavelength.

LITERATURE CITED

1. A. V. Lykov, Inzh.-Fiz. Zh., 26, No. 5, 781-793 (1974).
2. D. W. Phol and V. Irniger, Phys. Rev. Lett., 36, No. 9, 480-483 (1976).
3. D. H. Tsai and R. A. MacDonald, Phys. Rev., 14B, No. 10, 4714-4724 (1976).
4. H. Deresiewicz, J. Acoust. Soc. Am., 29, No. 2, 204-213 (1957).
5. Yu. K. Éngel'brekht, Izv. Akad. Nauk ESSR, Ser. Fiz.-Mat. Nauk, 22, No. 2, 188-195 (1973).
6. Ts. Ivanov and Yu. K. Éngel'brekht, Inzh.-Fiz. Zh., 35, No. 2, 344-351 (1978).
7. V. L. Kolpashchikov and S. Yu. Yanovskii, Inzh.-Fiz. Zh., 36, No. 6, 1093-1099 (1979).

SELF-SIMILAR SOLUTION OF THE PROBLEM OF CONSOLIDATION AND THAWING OF FROZEN SOIL

A. F. Klement'ev and E. A. Klement'eva

UDC 624.139.264:536.42

The article presents a new mathematical model of the process of thawing of frozen soil taking consolidation into account. The following solutions were obtained: the self-similar one for the unidimensional biphase problem and an approximate analytical one for the simplified single-phase problem.

In the calculation of engineering structures erected in soil massif that thaws under their thermal effect, it is usual to take into account the thermal regime of the buildings and the position of the boundary between thawed and frozen zones of the accommodating soil in dependence on time [1-3]. Thus it is implicitly assumed that the heat source is the engineering structure and that it is fixed. In reality, however, there occurs filtering consolidation of the thawing soil; as a result the heat source moves according to the law of increasing subsidence [4] which takes into account the variable thickness of the consolidating soil layer.

Bashkirian State University, Ufa.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 54, No. 4, pp. 662-665, April, 1988. Original article submitted January 9, 1987.

0022-0841/88/5404-0463$12.50

Thus, for the adequate prediction of thermal interaction of engineering structures with the thawing soil it is indispensable to solve the joint problem of consolidation and thawing of the soil.

To obtain the self-similar solution, we will examine the following simple mathematical model of this process.

Filtering consolidation of homogeneous thawing soil in the unidimensional case is described by the boundary-value problem:

$$\frac{\partial p}{\partial t} = c\frac{\partial^2 p}{\partial z^2}, \quad s(t) < z < \xi(t), \quad 0 < t \leqslant t_f; \tag{1}$$

$$p = 0, \quad z = s(t); \tag{2}$$

$$(p_0 - p)\frac{d\xi}{dt} = c\frac{\partial p}{\partial z}, \quad z = \xi(t); \tag{3}$$

$$p = p_0, \quad s = 0, \quad t = 0. \tag{4}$$

The subsidence of the thawing soil is found from the equation

$$s(t) = a_0 \int_{s(t)}^{\xi(t)} (p_0 - p)\,dz + A\xi(t). \tag{5}$$

The position of the boundary between the thawed and frozen zones is determined from the solution of Stefan's biphase problem for homogeneous soil with a view to the movement of the heat source:

$$\frac{\partial T_1}{\partial t} = a_1^2\frac{\partial^2 T_1}{\partial z^2}, \quad s(t) < z < \xi(t), \quad 0 < t < t_f; \tag{6}$$

$$\frac{\partial T_2}{\partial t} = a_2^2\frac{\partial^2 T_2}{\partial z^2}, \quad \xi(t) < z < \infty, \quad 0 < t < t_f; \tag{6.1}$$

$$T_1 = T_b \quad z = s(t); \tag{7}$$

$$T_1 = T_2 = T_m \quad z = \xi(t); \tag{8}$$

$$\lambda_1\frac{\partial T_1}{\partial z} - \lambda_2\frac{\partial T_2}{\partial z} = -Q\frac{d\xi}{dt}, \quad z = \xi(t); \tag{9}$$

$$T_2 = T_{pf} \quad z \to \infty; \tag{10}$$

$$T_1 = T_m \quad \xi = 0, \quad t = 0. \tag{11}$$

The self-similar solution of the joint problem of consolidation and thawing of frozen soil has the form

$$p = A_0 \operatorname{erf}\left(\frac{z}{2\sqrt{ct}}\right) + B_0, \tag{12}$$

$$T_s = A_s \operatorname{erf}\left(\frac{z}{2a_s\sqrt{t}}\right) + B_s, \quad s = 1,\ 2. \tag{13}$$

The regularities of subsidence of thawing soil and of the movement of the interface between the thawed and frozen zones are described by the expressions

$$s(t) = \beta\sqrt{t}, \tag{14}$$

$$\xi(t) = \alpha\sqrt{t}. \tag{15}$$

The coefficients α, β have to be determined. It is known that these regularities are confirmed with fairly high accuracy by experimental data [5].

The constants A_0, B_0, A_s, B_s (s = 1, 2) are determined from the boundary conditions:

$$A_0 = \frac{p_0}{P(\alpha_3) - \operatorname{erf}\beta_3}, \quad B_0 = -\frac{p_0 \operatorname{erf}\beta_3}{P(\alpha_3) - \operatorname{erf}\beta_3}; \tag{16}$$

$$A_1 = \frac{T_b - T_m}{\operatorname{erf}\beta_1 - \operatorname{erf}\alpha_1}, \quad B_1 = \frac{T_m \operatorname{erf}\beta_1 - T_b \operatorname{erf}\alpha_1}{\operatorname{erf}\beta_1 - \operatorname{erf}\alpha_1}; \tag{17}$$

$$A_2 = \frac{T_{pf} - T_m}{\operatorname{erfc}\alpha_2}, \quad B_2 = \frac{T_m - T_{pf}\operatorname{erf}\alpha_2}{\operatorname{erfc}\alpha_2}, \tag{18}$$

where $\alpha_s = \alpha/2a_s$, $s = 1, 2$; $\alpha_3 = \alpha/2\sqrt{c}$, $\beta_1 = \beta/2a_1$, $\beta_3 = \beta/2\sqrt{c}$, $P(\alpha_3) = \operatorname{erf}\alpha_3 + \frac{\exp(-\alpha_3^2)}{\alpha_3\sqrt{\pi}}$.

The coefficients α, β are determined from the following system of nonlinear equations:

$$\beta_3 = \frac{a_0 p_0 \beta_3 P(\beta_3)}{P(\alpha_3) - \operatorname{erf}\beta_3} + A\alpha_3, \tag{19}$$

$$(T_b - T_m)\frac{\lambda_1}{a_1}\frac{\exp(-\alpha_1^2)}{\operatorname{erf}\alpha_1 - \operatorname{erf}\beta_1} - (T_m - T_{pf})\frac{\lambda_2}{a_2} \times \frac{\exp(-\alpha_2^2)}{\operatorname{erfc}\alpha_2} = \frac{\sqrt{\pi}}{2} Q\alpha. \tag{20}$$

When we solve these equations jointly, it is not difficult in principle to find these coefficients and to determine from them the subsidence of the foundation of the building, the position of the interface between the thawed and the frozen zones, and also the vertical distribution of temperature and pore pressures at a specified instant.

However, in practical calculations it becomes necessary to use a computer for solving the system of nonlinear equations (19), (20). It is therefore of considerable interest to obtain approximate solutions not requiring the use of a computer, even if it is only for the simplified problem of consolidation and thawing of soils.

We will assume, as is usually done in the theory of filtering consolidation [4], that Eq. (1) may be dealt with in the region $0 < z < \xi(t)$, the boundary condition (2) for $z = 0$, and in the calculation of subsidence by formula (4) the lower limit of integration may be taken equal to zero. Then, in accordance with (16), we find

$$A_0 = \frac{p_0}{P(\alpha_3)}, \quad B_0 = 0. \tag{21}$$

We will determine the position of the interface between the thawed and frozen zones from Stefan's single-phase problem for homogeneous soil, with the movement of the heat source taken into account. In doing that, we determine the constants A_1, B_1 by formulas (17). The system of equations (19), (20) becomes considerably simplified and assumes the form

$$\beta_3 = \frac{a_0 p_0}{P(\alpha_3)} + A\alpha_3, \tag{22}$$

$$(T_b - T_m)\frac{\lambda_1}{a_1}\frac{\exp(-\alpha_1^2)}{\operatorname{erf}\alpha_1 - \operatorname{erf}\beta_1} = \frac{\sqrt{\pi}}{2} Q\alpha, \tag{23}$$

where the experimentally obtained [5] values of the thermophysical and deformation parameters $\exp(-\alpha_s^2)$ and $\operatorname{erf}\alpha_1$ can be replaced with an accuracy sufficient for engineering application by $1 - \alpha_s^2$ and $\sqrt{1.26}\,\alpha_1$, respectively. Then, after substituting (22) into (23), we obtain the biquadratic equation with respect to α_1:

$$b_1[1 + (1 - A)C_1]\alpha_1^4 + [1 - b_1 + (1 - a_0 p_0 - A)C_1]\alpha_1^2 - 1 = 0, \tag{24}$$

where

$$b_1 = 0.9896 a_1^2/c, \quad C_1 = Qa_1^2\sqrt{1.26\pi}/(\lambda_1 T_b).$$

There is no difficulty in solving this equation. From the calculated value of α_1 we can easily determine the subsidence of the foundation of the building, the position of the interface between the thawed and frozen zones, and also the vertical distribution of temperatures and pore pressures at a specified instant.

A comparison with the results of physical modeling showed that the suggested method is fairly effective in the case of "warm" permafrost. The mean error in predicting the position of the interface between the thawed and frozen zones for different soils over a period of 1-10 years amounted to 20.9%. The use of the method of the All-Union Research Institute of Pipeline Construction (VNIIST) yielded an error of 31.6%, and the method of the All-Union Research Institute of the Gas Industry (VNIIGaz) an error of 39.6%.

NOTATION

z, depth; t, time; t_f, final time; p, excess pore pressure; c, coefficient of consolidation; s, subsidence; ξ, depth of thawing; p_0, external load; a_0, coefficient of relative compressibility; A, coefficient of thawing; T_1, T_2, temperature of thawed and frozen soil, respectively; T_b, T_m, T_{pf}, temperature of the foundation of the building, melting point, temperature of the permafrost, respectively; a_1^2, a_2^2, thermal diffusivity of thawed and frozen soil, respectively; λ_1, λ_2, thermal conductivity of thawed and frozen soil, respectively; Q, heat of phase transition.

LITERATURE CITED

1. G. V. Porkhaev, Thermal Interaction of Buildings and Building Structures with Permafrost Soils [in Russian], Moscow (1970).
2. N. E. Khodanovich, B. L. Krivoshein, and R. N. Bikchentai, Thermal Regimes of Overland Gas Pipelines [in Russian], Moscow (1971).
3. B. L. Krivoshein, Thermophysical Calculations of Gas Pipelines [in Russian], Moscow (1982).
4. N. A. Tsytovich, The Mechanics of Frozen Soils [in Russian], Moscow (1973).
5. S. S. Vyalov and G. M. Pakhomova, in: Thawings Soils as Foundations of Buildings [in Russian], Moscow (1981), pp. 47-55.